

Kinks, extra dimensions, and gravitational waves

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ABSTRACT: We investigate in detail the gravitational wave signal from kinks on cosmic (super)strings, including the kinematical effects from the internal extra dimensions. We find that the signal is suppressed, however, the effect is less significant than that for cusps. Combined with the greater incidence of kinks on (super)strings, it is likely that the kink signal offers the better chance for detection of cosmic (super)strings.

KEYWORDS: Large Extra Dimensions, Cosmic Superstrings.

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1. Introduction

Cosmic strings, [1], continue to play an important role in the diagnostic testing of high energy physics in the early universe. Initially, these strings were seen as relic topological defects of some Grand Unified phase transition in the early universe, [2], and it was hoped that they would provide a causal alternative to inflation for the generation of primordial perturbations. However, it was rapidly realised that the perturbation spectrum predicted by strings was in contradiction to the microwave background measurements, [3], and for a while, cosmic strings were relegated to the “also-rans” of early universe cosmology. Recently however, cosmic strings have come back to the fore as by-products of the brane inflationary scenario in string theory [4, 5, 6, 7, 8].

The basic cosmological picture is that cosmic strings arise during some phase transition in the early universe, [2]. By their nature, they are topologically stable, and can therefore persist as a network of loops and long string. From the cosmological point of view, the internal structure of the very thin string is irrelevant, and it is modelled by a zero thickness line-like object whose motion is determined by the Nambu action, [9, 10]:

$$S = -\mu \int d^2\sigma \sqrt{\gamma} \tag{1.1}$$

where μ is the mass per unit length of the string, and γ the induced metric on the worldsheet. Together with rules for intercommutation [11], or how crossing strings interact, this gives the basic physics of how a network of cosmic strings will evolve. Incorporating gravitational effects via a linearised approximation indicates how fast energy is lost from the network, [12], and putting all these pieces together gives the scaling picture of the original cosmic string scenario [13].

Cosmic (super)strings¹, [14], are generally modelled in a similar fashion to cosmic strings, but with one or two key differences arising because they are derived from a higher dimensional theory, string theory, and have additional physics arising as a result. As was realised early on, these (super)strings need not intercommute when they apparently intersect in our 4D world, since they can miss each other in the internal dimensions [15], resulting in a denser network, [16]. Another key difference is that the (super)strings can move in the internal dimensions, thus altering the effective kinematics.

It is the effect of these extra dimensions on the kinematics, and their consequences that we are interested in. The Nambu action leads to a simple set of equations whose solutions are left and right moving waves along the string, [17]. Mostly, the (super)string moves in a nonsingular, albeit highly relativistic, fashion. However, there are two main exceptions: cusps and kinks². Cusps are transitory events, where constructive interference between the left and right moving modes causes a point of the (super)string to instantaneously reach the speed of light. Kinks on the other hand are relics of intercommutation, they are sharp ‘corners’ on the (super)string and represent a discontinuity in the wave velocity of either a left or right mover; because they are a feature of only one wavefront, they persist and move along the (super)string, [20]. Since both kinks and cusps represent a certain level of singularity of the world-sheet swept out by the (super)string, when including gravitational backreaction they generate strong and distinct gravitational wave signals, [12, 21, 22, 23, 24, 25].

Typically, gravitational wave emission from (super)string loops is dominated by low frequency radiation, and contributes to the decay process of loops and the general gravitational wave background. However, in an important paper, Damour and Vilenkin (DV), [24], showed that for cusps and kinks, the high frequency radiation was not exponentially damped, but had a significant power law tail, and hence a significant burst of power. They showed how to compute the gravitational waveform, and that the cusp/kink had a characteristic beaming pattern in which radiation was narrowly focussed into a cone/fan associated with the wavevectors of the cusp/kink respectively.

In recent papers, [26], we explored the impact of the extra internal dimensions for the (super)string motion. As first pointed out in [27], these “slow down” the (super)string from our 4D perspective: this obviously has an impact on the definition and likelihood of a cusp, which requires that the (super)string instantaneously moves at the speed of light. In [26] we showed how these internal degrees of freedom alter the

¹We use the conventional bracketed (super) to indicate that these strings are neither genuine superstrings, nor old fashioned cosmic strings living in four dimensions (4D), but are some approximate line like object with some number of effective extra internal dimensions in which they can move.

²Certain cosmic (super)strings can also have junctions, [15, 18, 19], however we will not consider this additional feature in detail here.

gravitational wave signal from cusps; the effect proved to be significant: contrary to previous expectations, [8, 25, 28], the gravitational radiation signal is not enhanced by the lowering of the intercommutation probability in extra dimensions, but is in fact suppressed by several orders of magnitude, mainly due to a probabilistic effect from the altered kinematics. We had initially focussed on cusps as they were expected to give a much stronger signal than kinks, but clearly, with the cusp signal so suppressed, a re-visiting of the kink signal is in order.

The key features of the kink are its sharp nature and persistence. As shown by Damour and Vilenkin (DV), [24], (see also [21]) a kink gravitational wave burst (GWB) looks like a “fan”, i.e. strongly localised in one angular direction, but spreading out in the other. Thus, although the high frequency tail of the GWB has a stronger fall-off than the cusp, it is radiated over a greater solid angle and is therefore still relevant. Moreover, if, as argued in [29], kinks are prolific on cosmic (super)strings, it is possible that the effect of proliferation can lift the kink signal above that of the cusp. Finally, since kinks only require a discontinuity in the wave velocity, and since cosmic (super)strings still intercommute, albeit with a reduced probability, the mechanism for kink formation would appear to be robust, and thus some of the probabilistic suppression that we saw in the case of cusps may not be a factor in the case of kinks.

In this paper we calculate how the extra dimensions affect the kink signal. The impact of including the internal degrees of freedom is to narrow the spread of the “fan”; there is no corresponding parameter space reduction of the kink probability for the reasons mentioned above. Thus, the damping of the kink signal is considerably less pronounced than that of the cusp, and our conclusion is that it is the kink, and not the cusp, GWB’s that are most likely to be seen by LIGO or LISA. We also look carefully at the geometry of the kink burst by analysing its magnitude, and show that it is a genuine fan, in that it has finite angular extent in both directions, although is less sharply cut-off in its extended direction. As a by-product of our analysis, we correct the approximations used thus far in the literature for the amplitude, [29, 30].

2. Cosmic string kinks in four dimensions

We briefly review standard cosmic string kinematics from the formalism developed by Kibble and Turok, [17], then the DV calculation of the kink GWB. Finally, we discuss the amplitude and geometry of the kink GWB and give an analysis of the actual contribution to the GW signal from a generic kink.

Writing the spacetime coordinates of the string as $X^\mu(\sigma^A)$, where $\sigma^A = \{\tau, \sigma\}$ are intrinsic coordinates on the worldsheet, with $\sigma \in [0, L]$, for closed loops of length L . Kibble and Turok chose a gauge with conformally flat worldsheet metric, centre of mass spacetime coordinates, and worldsheet time corresponding to coordinate time. Writing $X^\mu = (\tau, \mathbf{r}(\tau, \sigma))$, they found the solutions to the equation of motion for the

worldsheet take the form of left and right moving waves, conventionally written in the form

$$\mathbf{r} = \frac{1}{2}[\mathbf{a}(\sigma_-) + \mathbf{b}(\sigma_+)], \quad (2.1)$$

where $\sigma_{\pm} = \tau \pm \sigma$ are lightcone coordinates, and the gauge conditions constrain \mathbf{a}' and \mathbf{b}' to lie on a unit sphere, commonly dubbed the “Kibble-Turok” sphere:

$$\mathbf{a}'^2 = \mathbf{b}'^2 = 1. \quad (2.2)$$

Notice that while the periodicity of \mathbf{a} and \mathbf{b} is L , the periodicity of the actual motion of the string is $L/2$, since $\mathbf{r}(\sigma + L/2, \tau + L/2) = \mathbf{r}(\sigma, \tau)$. There is also an additional constraint that $\langle \mathbf{a}' \rangle = \langle \mathbf{b}' \rangle = 0$, coming from the facts that the loop is closed and that we are in the *c.o.m.* frame. Thus the string motion is specified by two curves on the unit sphere which must have zero weight.

Kinks, unlike cusps, which occur whenever these curves intersect, form when one of \mathbf{a}' , \mathbf{b}' or one of their derivatives has a discontinuity. As well as being ubiquitous on cosmic strings, kinks can also prevent the formation of cusps on cosmic string loops, as a discontinuity on either \mathbf{a}' or \mathbf{b}' allows the curves to miss each other if they would otherwise have intersected at that point. Kinks, like cusps, emit bursts of gravitational radiation whose signal was calculated initially by Vachaspati and others, [21, 31], and more rigorously by DV, [24], as we now briefly review.

In order to obtain the gravitational waveform of the kink GWB, we begin with the linearised Einstein equations

$$\square \bar{h}_{\mu\nu} = -16\pi G T_{\mu\nu} \quad (2.3)$$

where $\bar{h}_{\mu\nu}$ is the trace reversed metric perturbation, and $T_{\mu\nu}$ is the string energy momentum

$$T^{\mu\nu}(\mathbf{k}, \omega) = \frac{\mu}{L} \int_0^L d\sigma_+ \int_0^L d\sigma_- \dot{X}_+^{(\mu} \dot{X}_-^{\nu)} e^{-\frac{i}{2}(k \cdot X_+ + k \cdot X_-)} \quad (2.4)$$

written in momentum space with $k^\mu = \frac{4\pi m}{L}(1, \mathbf{n}) = m\omega_L(1, \mathbf{n})$, where ω_L is the frequency of the fundamental mode of the string loop, and X_{\pm} denote the position vectors $X_{\pm}^\mu = (\sigma_{\pm}, \mathbf{a}/\mathbf{b}(\sigma_{\pm}))$ associated with the left and right moving modes ($X^\mu = (X_+^\mu + X_-^\mu)/2$). In the far field approximation the solution is given by

$$\bar{h}_{\mu\nu} \simeq \frac{4G}{r} \sum_{\omega} e^{-i\omega(t-r)} T_{\mu\nu}(\mathbf{k}, \omega), \quad (2.5)$$

thus finding the perturbation reduces to determining the integrals

$$I_{\pm}^\mu = \int_0^L d\sigma_{\pm} \dot{X}_{\pm}^\mu e^{-\frac{i}{2}k \cdot X_{\pm}} \quad (2.6)$$

which appear in the energy momentum tensor.

The key first step of the DV calculation is to determine when these integrals are not exponentially suppressed, and to remove the leading order gauge dependence. Without loss of generality in what follows, we will assume that the discontinuity is in the \mathbf{a}' curve, and that the \mathbf{b}' curve is continuous.

First, note that around σ_+ , we can expand the right moving position vector X_+^μ as

$$X_+^\mu(\sigma_+) = \ell^\mu \sigma_+ + \frac{1}{2} \ddot{X}_{0+}^\mu \sigma_+^2 + \frac{1}{6} \ddot{X}_{0+}^\mu \sigma_+^3 \quad (2.7)$$

where the subscript 0 refers to evaluation at $\sigma_+ = 0$. Defining the angle between k^μ and ℓ^μ as θ , which is assumed to be small, and writing $d^\mu = k^\mu - \ell^\mu = (0, \mathbf{d})$ (where $|\mathbf{d}| \simeq \theta$), and using the gauge conditions, DV obtain:

$$k_\mu X_+^\mu = m\omega_L \left[\frac{1}{2} \theta^2 \sigma_+ - \frac{1}{2} \mathbf{n} \cdot \mathbf{b}'' \sigma_+^2 + \frac{1}{6} (\mathbf{b}''^2 - \mathbf{d} \cdot \mathbf{b}''') \sigma_+^3 \right] \quad (2.8)$$

(although they drop the $\mathbf{d} \cdot \mathbf{b}'''$ term as it is subdominant). Thus the integral takes the form

$$I_+^\mu = \int [k^\mu - d^\mu + \ddot{X}_+^\mu \sigma_+] \exp \left[-\frac{im\omega_L}{12} (3\theta^2 \sigma_+ - 3\theta |\ddot{X}_+|^2 \sigma_+^2 \cos \beta + |\ddot{X}_+|^2 \sigma_+^3) \right] d\sigma_+ \quad (2.9)$$

where β is the angle between \mathbf{d} and \mathbf{b}'' . As Damour and Vilenkin pointed out, the first k^μ term is a pure gauge, (although there are subtleties when $d^\mu \neq 0$) and therefore the physical part of the integral is proportional to \ddot{X}_+^μ .

For $\theta = \beta = 0$, this integral can be evaluated analytically (see also [26]) as

$$I_+^\mu = - \left(\frac{12}{m\omega_L \ddot{X}_+^2} \right)^{2/3} \frac{i}{\sqrt{3}} \Gamma \left(\frac{2}{3} \right) \ddot{X}_+^\mu. \quad (2.10)$$

Note that the origin of σ_+ is arbitrary, hence “ $\theta = 0$ ” means that the wave vector coincides with the velocity of the right mover for some value of σ_+ , indeed a shift in origin of σ_+ simply corresponds to introducing a disparity $d^\mu = -\delta\sigma_+ \ddot{X}_{0+}^\mu$, thus the “ $\beta = 0$ ” saddle is simply reflecting this gauge invariance.

We therefore arrive at the conclusion that the contribution to the gravitational perturbation for the (continuous) right moving mode has a power law decay of $f^{-2/3}$, and is strongly localised around the velocity vector for that mode. As shown in DV the angular spread transverse to the \mathbf{b}' trajectory is

$$\theta_m = \left(\frac{12\ddot{X}}{m\omega_L} \right)^{1/3} \simeq \left(\frac{2}{Lf} \right)^{1/3}. \quad (2.11)$$

Thus the integrals I_\pm are enhanced for wave vectors coincident with the wave on the string. Clearly, when these left and right moving vectors coincide, this corresponds to both I_+ and I_- being on their saddle points which gives a strong signal sharply

localised in a solid angle $\pi\theta_m^2$ around this direction. However, since we are considering a kink, we must assume that the discontinuity causes the \mathbf{a}' curve to ‘miss’ the \mathbf{b}' curve, and thus we must look more closely at I_- , which requires a slightly different approach.

To compute the kink integral, DV split it into three parts around the discontinuity at $\sigma_- = \sigma_-^{disc}$, taking \dot{X}_-^μ to jump from $n_1^\mu = (1, \mathbf{n}_1)$ at $\sigma_-^{disc} - 0$ to $n_2^\mu = (1, \mathbf{n}_2)$, at $\sigma_-^{disc} + 0$ where $\mathbf{n}_{1/2}$ are unit vectors. The integral becomes:

$$I_-^\mu = \int_{\sigma_0}^{\sigma_-^{disc}-\epsilon} d\sigma_- \dot{X}_-^\mu e^{-\frac{i}{2}(k \cdot X_+ + k \cdot X_-)} + \int_{\sigma_-^{disc}-\epsilon}^{\sigma_-^{disc}+\epsilon} d\sigma_- \dot{X}_-^\mu e^{-\frac{i}{2}(k \cdot X_+ + k \cdot X_-)} + \int_{\sigma_-^{disc}+\epsilon}^{\sigma_0+L} d\sigma_- \dot{X}_-^\mu e^{-\frac{i}{2}(k \cdot X_+ + k \cdot X_-)}, \quad (2.12)$$

where ϵ is an arbitrary small parameter, introduced to separate out the discontinuity from the rest of the integral (i.e. the middle term of (2.12)), which will be allowed to go to zero at the end of the calculation. The first and third terms then cover the remainder of the integration range on either side of the discontinuity.

The terms in (2.12) which no longer contain the discontinuity only have a significant contribution when the wavevector is parallel to the velocity vector, however, by assumption, this will not correspond to a saddle point of the I_+ integral, otherwise we would have a double saddle and hence a cusp. Thus we must take k^μ to lie outside the range $\pm\theta_m$ of the wave vector \dot{X}_-^μ , and therefore the first and third terms in eqn. (2.12) can be neglected. Eqn. (2.12) can therefore be evaluated to leading order as

$$I_-^\mu \simeq \int_{\sigma_-^{disc}-\epsilon}^{\sigma_-^{disc}} n_1^\mu e^{-\frac{i m \omega_L}{2} \hat{k} \cdot n_1 \sigma_-} - \int_{\sigma_-^{disc}+\epsilon}^{\sigma_-^{disc}} n_2^\mu e^{-\frac{i m \omega_L}{2} \hat{k} \cdot n_2 \sigma_-} \quad (2.13)$$

$$= -\frac{2n_1^\mu}{i m \omega_L \hat{k} \cdot n_1} e^{-\frac{i m \omega_L}{2} \hat{k} \cdot n_1 \sigma_-} \Big|_{\sigma_-^{disc}-\epsilon}^{\sigma_-^{disc}} - \left(-\frac{2n_2^\mu}{i m \omega_L \hat{k} \cdot n_2} \right) e^{-\frac{i m \omega_L}{2} \hat{k} \cdot n_2 \sigma_-} \Big|_{\sigma_-^{disc}+\epsilon}^{\sigma_-^{disc}} \\ \simeq \frac{2i}{m \omega_L} \left(\frac{n_1^\mu}{\hat{k} \cdot n_1} - \frac{n_2^\mu}{\hat{k} \cdot n_2} \right) = \frac{2i}{m \omega_L} v^\mu \quad (2.14)$$

where $\hat{k} = (1, \mathbf{n})$, so that $m \omega_L \hat{k}^\mu = k^\mu$.

Putting together the I_\pm integrals, DV argued that the logarithmic kink waveform, $h^{\mu\nu}(f) = 2G\mu|f|I_+^{(\mu)}I_-^{(\nu)}/r$, is therefore

$$h^{kink}(f, \theta) \sim \frac{G\mu L^{1/3}}{r|f|^{2/3}} H[\theta_m - \theta]. \quad (2.15)$$

To transform (2.15) to a cosmological setting, we replace f by $(1+z)f$, where z is now the redshift at the time of emission of the kink burst, and r by the physical distance

$$a_0 r = a_0 \int_{t_e}^{t_0} \frac{dt}{a} = \int_0^z \frac{dz}{H} = (1+z)D_A(z) \quad (2.16)$$

where $D_A(z)$ is the angular diameter distance at redshift z .

To estimate the GWB signal rate from the cosmological network, DV assumed the one scale model, in which the network is approximated by the dominant scale set by the cosmological time:

$$L \sim \alpha t \quad , \quad n_L(t) \sim 1/(\alpha t^3) \quad (2.17)$$

for the length and number density of the string network at cosmological time t . Here $\alpha \sim \Gamma G\mu$ is a numerically determined constant, [12, 22, 23, 32], presumed to represent the rate of energy loss from string loops via gravitational radiation, where, as in DV, we take $\Gamma \sim 50$.

Finally, the rate of GWB's from kinks observed in the spacetime volume in redshift interval dz around a frequency f can be calculated from

$$d\dot{N}_k = \frac{2\nu_k(z)}{1+z} \frac{\pi\theta_m D_A(z)^2}{(1+z)H(z)} dz \quad , \quad (2.18)$$

where the number of kink events on a loop ν_k is given by

$$\nu \sim \mathcal{K} \frac{n_L}{PT_L} \sim \frac{2\mathcal{K}}{P\alpha^2 t^4} \quad (2.19)$$

with \mathcal{K} being the average number of kinks per loop.

In order to perform this calculation, DV made several simplifications concerning the kink GWB which we will now critically examine. First of all, in deriving the kink GWB amplitude (2.15) the amplitude of the integral I_- was taken to be $\mathcal{O}(1)$.

In order to get a feel for the kink signal, consider the magnitude of v^μ in (2.14)³. Choosing coordinates for the Kibble-Turok 2-sphere so that the relevant kink and gravitational wave 3-vectors are:

$$\begin{aligned} \mathbf{n}_1 &= \cos \frac{\delta}{2} \mathbf{e}_1 + \sin \frac{\delta}{2} \mathbf{e}_2 \\ \mathbf{n}_2 &= \cos \frac{\delta}{2} \mathbf{e}_1 - \sin \frac{\delta}{2} \mathbf{e}_2 \\ \mathbf{n} &= \sin \theta \cos \phi \mathbf{e}_1 + \sin \theta \sin \phi \mathbf{e}_2 + \cos \theta \mathbf{e}_3 \end{aligned} \quad (2.20)$$

where δ is the *discontinuity angle*, i.e. the angle between \mathbf{n}_1 and \mathbf{n}_2 . The ‘amplitude’ of the kink can therefore be straightforwardly calculated as:

$$\mathcal{A} = |v^\mu v_\mu|^{1/2} = \frac{2 \sin \delta/2}{[1 - \sin \theta \cos(\phi + \delta/2)]^{1/2} [1 - \sin \theta \cos(\phi - \delta/2)]^{1/2}}. \quad (2.21)$$

Note that this differs from the definition of the amplitude in [29, 30] in two ways. First of all, we have taken the magnitude of the 4-vector v^μ , this is important because

³Recall that we have already used gauge invariance to remove the k^μ factor in I_+ .

the worldsheet is a relativistic object, and the gravitational radiation is determined by the energy momentum tensor. The 4-vector component is in fact related to terms appearing in the extrinsic curvature of the worldsheet at the kink, (see appendix of [26]), and is thus the appropriate relativistic quantity to consider. In addition, [29] explicitly removed the denominator from consideration, arguing that it was $\mathcal{O}(1)$. While this is generally true, it is equally true of the numerator in v^μ , and both contribute to the overall amplitude. We therefore arrive at the result that the amplitude varies by more than an order of magnitude over the sphere, and, depending on the θ_m cutoff, potentially significantly more.

By plotting the density of \mathcal{A} on the unit sphere and combining with the saddle of the \mathbf{b}' curve, the gravitational radiation from the kink can be seen to be spread out along the \mathbf{b}' curve, but does peak significantly in the region where that curve runs between the two points marking the kink discontinuity. By supposition, the curve must run between the discontinuity (otherwise a cusp would result) as it must have zero weight, $\langle \mathbf{b}' \rangle = \mathbf{0}$.

In figure 1 the amplitude is shown along a great circle running between the discontinuity for a range of discontinuity angles. We have chosen a generic great circle which peaks at a latitude of 30° , and crosses the equator at a longitude of $18^\circ E$. The figure shows both the large range in the amplitude \mathcal{A} , as well as the focussing along the curve. For example, the black curve, corresponding to a discontinuity angle of $\delta = \pi/3$, is sharply peaked between $\lambda \sim 3\pi/8 \rightarrow 5\pi/8$ corresponding to a genuine fan of radiation. The figure also shows clearly that as the discontinuity angle increases, \mathcal{A} decreases between the discontinuity points, and thus contrary to [29], we find that the *closer* these points are, the more strongly focussed the gravitational radiation and the more power it has.

The second simplification DV make is in the solid angle integration of the GWB rate (2.18). Here, the integral of the kink GWB is assumed to cover a solid angle $2\pi\theta_m$. This reflects the integration over the fan, which in this instance is approximated by a great circle (or the fundamental mode). Since the kink signal peaks for wavevectors in the region between the discontinuity, the actual solid angle of the fan is less than $2\pi\theta_m$.

In order to confirm that these effects do not significantly change the DV result, we took various weighted averages of the kink signal. First, the most straightforward, we simply integrated the amplitude over the sphere to find an average amplitude per unit solid angle. This varies remarkably little as the discontinuity angle varies. However, as the \mathbf{b}' curve must run between the \mathbf{n}_i , a more accurate measure might be to take into account the localisation of the GWB on the fan, and average over all possible “fans” running between the discontinuity. Using great circle trajectories such as those in figure 1, the result is again a mild dependence on the discontinuity angle, but an anti-correlation in this case. The clear outcome is that the weighting of the integral due to the amplitude and geometry of a kink is indeed well approximated

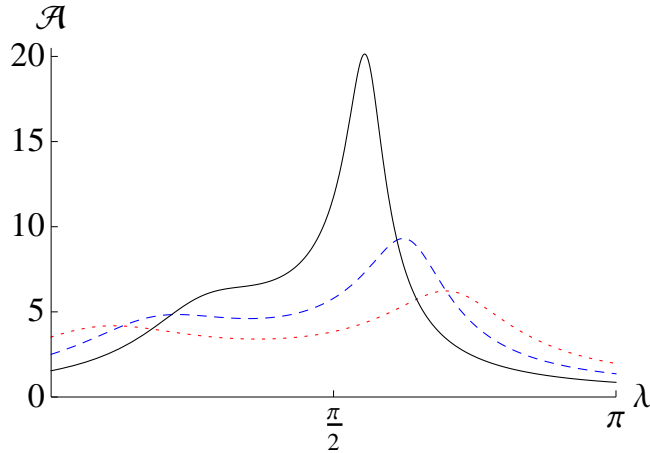


Figure 1: The kink amplitude (2.21) along a great circle for various angles of separation for the discontinuity as a function of the proper distance along the path. In each case the great circle runs from $\theta = \pi/3$ to $2\pi/3$, and crosses the equator at $\phi = \pi/10$. The dotted red line corresponds to a discontinuity angle of $\delta = 2\pi/3$, the dashed blue line to $\delta = \pi/2$, and the solid black line to $\delta = \pi/3$.

by a number of $\mathcal{O}(1)$ in a solid angle of $\mathcal{O}(\pi\theta_m)$.

To recap: the DV computation shows due to the mobile and persistent nature of a kink, the emission is not beamed in a cone like the cusp, but is instead radiated in a one dimensional ‘fan-like’ set of directions, [24], which arise from the occurrence of saddle points; the inclusion of the angle θ_m , so that the observation direction is slightly offset from the direction of the emission, results in the 1D ‘line’ of emission from the kink being widened so that a strip is swept out on the surface of the unit sphere and the volume factor becomes $\theta_m/2$, [33].

3. Kinks in Higher Dimensions

The effect of extra dimensions on the formation of, and gravitational radiation from, cusps was explored in [26], where it was found that one of the main differences in looking at cusps with extra dimensions was the fundamental change in behaviour of 2D curves moving on the surface of a $2+n$ -sphere rather than a 2-sphere, resulting in intersections, which had been generic, becoming extremely rare. A kink, however, does not require the curves to intersect, nor will the appearance of the discontinuity depend on the number of dimensions in which the (super)string moves; kink formation will thus be unaffected by the inclusion of extra dimensions.

This is not to say that the presence of extra dimensions will have no effect on the GWB’s emitted by kinks. The change in the intercommutation probability due to the presence of extra dimensions, [15], affects the network density, leading to an enhancement of the kink amplitude. Furthermore, the slowing down of the wave

velocity in our noncompact dimensions affects the computation of the saddle point integral, I_+ , which will affect the kink GWB as it does the cusp, where it contributed to a reduction in the radiation cone, [26].

Following the notation of [26], we consider (super)string solutions in $4 + n$ dimensions, which can be expressed in Kibble-Turok notation as

$$\mathbf{R} = \frac{1}{2}[\mathbf{A}(\sigma_-) + \mathbf{B}(\sigma_+)], \quad (3.1)$$

where the upper case denotes the full $(3 + n)$ dimensional spatial vectors and $|\mathbf{A}'|^2 = |\mathbf{B}'|^2 = 1$ from the gauge choice as before. Since a kink simply requires a discontinuity in one of the left- or right- moving curves, the fact that the three dimensional part of these (still written as \mathbf{a}', \mathbf{b}') no longer lies on a unit S^2 has no relevance for kink formation in the extra dimensions. The computation of the I_- integral, dominated by the endpoints of the discontinuity, will also be unaltered.

However, the saddle point integral I_+ will damp too quickly unless we take the extra dimensional component, $b \ll 1$, where $|\mathbf{b}'|^2 = 1 - b^2$. Combining this with the estimation of $\mathbf{n}' \cdot \mathbf{b}'' = \mathcal{O}(b)|\ddot{X}|$ following the methods of [26], the exponent of I_+ is:

$$k_\mu X_+^\mu = \frac{1}{2}(\theta^2 + b^2) \sigma_+ - \frac{1}{2}(\theta + b) |\ddot{X}_+| \sigma_+^2 + \frac{1}{6} \ddot{X}_+^2 \sigma_+^3, \quad (3.2)$$

thus the angle θ_m narrows as a result of the inclusion of extra dimensions, which will affect the kink GWB signal.

We therefore obtain that, as for the cusp [26], the extra dimensional (logarithmic) kink waveform is the same as the 3D waveform, eqn. (2.15), with a reduced beaming angle:

$$h^{kink}(f, \theta) \sim \frac{G\mu L^{1/3}}{r|f|^{2/3}} H[\theta_b - \theta], \quad (3.3)$$

where

$$\theta_b = \theta_m - b \simeq \left(\frac{2}{Lf} \right)^{1/3} - b. \quad (3.4)$$

We now need to compute the event rate of these higher dimensional kink GWB's for the cosmological (super)string network. Since the mechanism which produces kinks is unchanged by the inclusion of extra dimensions, we assume that the average number of kinks in a loop, \mathcal{K} , is unchanged in the higher dimensional setting and will not depend on the number of extra dimensions, (unlike cusps as discussed in [26]). However, the (super)string network will possess kinks with a range of different θ_b values; we therefore write

$$\frac{d\dot{N}_k}{dz db} \sim \frac{2\mathcal{K}n_L(z)}{PT_L(z)} \frac{\pi(\theta_m(z) - b) D_A(z)^2}{(1+z)^2 H(z)}. \quad (3.5)$$

Integrating over b thus yields the kink GWB rate

$$\frac{d\dot{N}_k}{dz} \sim \frac{\mathcal{K}\theta_m^2 n_L(z)}{PT_L(z)} \frac{\pi D_A(z)^2}{(1+z)^2 H(z)}. \quad (3.6)$$

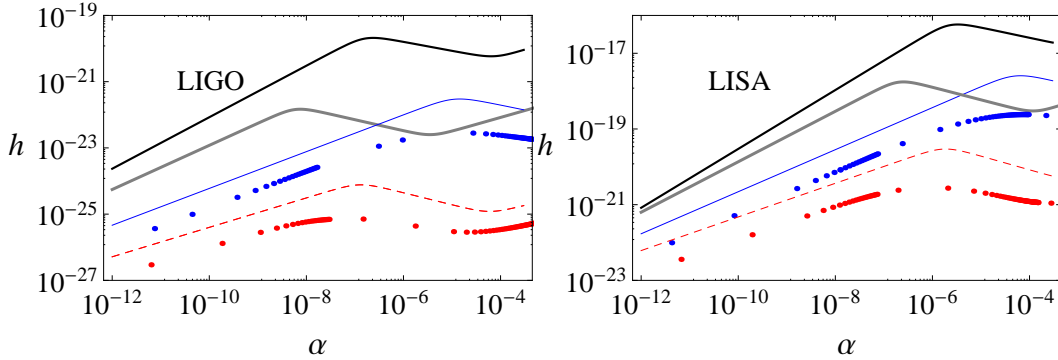


Figure 2: Log-log plots of the GWB amplitude as a function of α for the LIGO ($f = 150$ Hz) and LISA ($f = 3.9$ mHz) frequencies at a detection rate of 1 per year. The original DV computation is shown as a solid blue line, and the kink amplitude with extra dimensions is shown as a dashed red line. The sets of individual dots correspond to the exact numerical redshift integrations. For comparison, the cusp amplitude computed using interpolating functions is also shown in solid thick black for no extra dimensions, and thick grey for one additional extra dimension. All plots use an intercommutation probability of $P = 10^{-3}$.

Figure 2 shows the gravitational wave amplitude for the cosmic string kink burst calculated by DV, [24], along with our higher dimensional results for the characteristic frequencies of the LIGO and LISA gravitational wave detectors. We present the amplitudes calculated using the DV interpolating functions (neglecting Ω_Λ), and also include the results of an exact numerical integration performed for the concordance cosmology ($\Omega_r = 4.6 \times 10^{-5}$, $\Omega_m = 0.28$, $\Omega_\Lambda = 1 - \Omega_m - \Omega_r$). For comparison, we also include the equivalent plots for the cusp, with the amplitudes for zero and one extra dimension shown. A fixed value, $P = 10^{-3}$ was chosen for all the plots. Note that there is no dependence on the number of extra dimensions with the kink amplitude, but for cusps, with each additional extra dimension, the amplitude is suppressed by approximately one order of magnitude.

An alternative useful way of displaying the kink GWB is to instead plot the expected rates of detection at a given amplitude as described in [34]. The one scale model is something of an oversimplification (though the only sensible analytic approximation to date), and in [34], the authors show how to have full dynamical range for the network, including a dependence in h on the length distribution in the network, giving a rate per unit redshift, per unit amplitude. Integrating out using the one scale model, which essentially associates redshift with amplitude, gives an expected detection rate at a particular amplitude. Figure 3 shows this rate calculation for the kinks with extra dimensions in the frequency range of LIGO and LISA. In this case, the amplitude of the kink waveform is found by comparison with

$$h^{kink} = A|f|^{-2/3}, \quad (3.7)$$

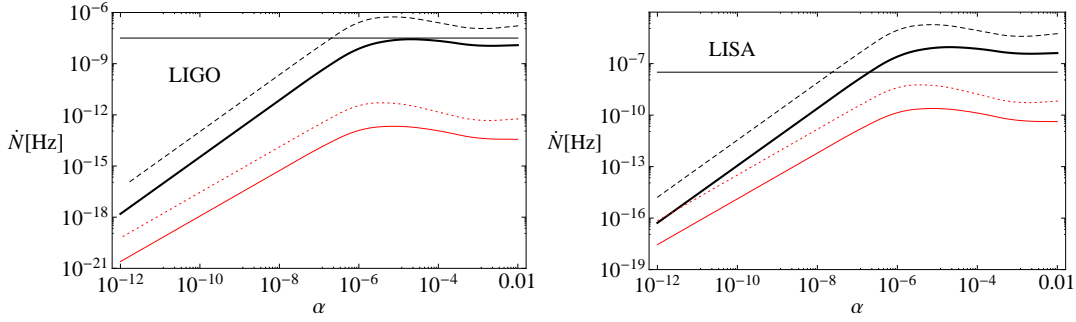


Figure 3: Similar plots to those of figure 2 but in this case showing the expected rate at an amplitude of $10^{-21} \text{ s}^{-1/3}$, using the method of [34]. In this case, the thick solid lines are the numerical integration results. From top to bottom: the DV kink, (black, solid) and the extra dimensional kink, (red, dashed). Both have an intercommutation probability of $P = 10^{-3}$. The horizontal black line indicates a rate of one event per year.

so that the redshift values used for the integration at various values of $G\mu$ are found from

$$\frac{\varphi_t^{1/3}(z)}{(1+z)^{2/3}\varphi_r(z)} = \frac{50AH_0^{-2/3}}{\alpha^{4/3}}, \quad (3.8)$$

where we once again use $A = 10^{-21} \text{ s}^{-1/3}$, $\alpha \sim 50G\mu$ and φ_t, φ_r are either the DV interpolating functions or related to the exact functions $t(z)$, $D_A(z)$, (c.f. Siemens et al., [34]).

4. Discussion

We have computed the gravitational wave signal due to cosmic (super)string kinks, and have found that there is a dimension independent suppression of approximately an order of magnitude compared to the purely four-dimensional result. The fact that the suppression is independent of the number of internal dimensions is in contrast to the cusp signal, [26], which drops sharply with each extra internal dimension. The reason for this difference is that the cusps have an additional probabilistic suppression due to the factors determining their formation.

Motion in the extra dimensions slows down the (super)string in the non-compact dimensions, and for the cusp, which requires that the (super)string reach the speed of light due to a constructive interference effect, this has a strong impact on the likelihood of formation. Kinks in contrast require only a discontinuity in the wave-vectors of the (super)string, and will persist even if those wave-vectors are not quite null. Moreover the mechanisms for kink formation (self-intersection, intercommutation) are not qualitatively affected by extra dimensions, thus the (super)string network will still have many kinks. Although the kink signal is at least an order of magnitude lower than the cusp signal before the extra dimensions are taken into account, the

ubiquitous nature of kinks on a (super)string loop in extra dimensions, as compared to the near cusp events discussed in [26], makes the kink amplitude an important signal, particularly if the number of extra dimensions is large. A particularly important point to note is that, like DV, we have assumed a value of $\mathcal{K} \simeq 1$. Clearly this is a gross underestimate of the true gravitational signal, the signal rises approximately one order of magnitude for every three orders of magnitude increase in \mathcal{K} . If kinks indeed proliferate on (super)strings, the real picture may in fact be extremely optimistic for (super)string detection.

A by-product of our analysis was that in computing the kink amplitude in section 2, we carefully followed the magnitude of the discontinuous wave integral I_- . Previously in the literature, [30, 29], the amplitude had been defined by taking the 3-vector and ignoring a dividing factor. By using the full relativistic expression, and keeping all factors, we showed that the actual picture for the kink amplitude was rather different. In contrast to [29], we find that very ‘sharp’ (where sharp refers to the three dimensional profile and corresponds to a large discontinuity angle) kinks do not radiate more than apparently milder kinks.

It is worth explaining why this is so, as it can seem counter-intuitive. The real key to gravitational radiation is the relativistic nature of the source. Large discontinuity kinks move slowly along the (super)string, and thus although they are sharp in the three dimensional sense, they do not move fast. In fact, the less ‘sharp’ kinks move much faster, hence radiate more. Of course, this modelling is looking at an individual kink, in reality, the loops will have many kinks, and there may also be many sharp kinks moving rapidly. However, we believe this is best reflected in an increase in the overall GWB amplitude via \mathcal{K} , rather than in the individual kink burst.

We have also not explored the effect of junctions in detail. Cosmic (super)strings have the additional possibility of having junctions due to the microphysics inherited from the underlying type IIB string theory [35]. These junctions can effect the network evolution, [36], as well as having gravitational radiation of their own [18]. Most recently, it has been shown that the kinematics of the junctions [37] can lead to a proliferation of kinks, [29], as a kink hitting a junction both reflects and transmits through the junction. All of these effects indicate that kinks are possibly a far more important feature of (super)string networks than the ordinary type of cosmic string.

It is worth noting that the effects of extra dimensions are dependent on the frequency at which they are observed; it is evident, (particularly from the rate plots in figure 3), that there is a much better chance of observing a kink signal at LISA rather than LIGO, although the approximation used in this method to find the waveform, (i.e. that $\theta_m \ll 1$), breaks down at nHz frequencies, where the angle $\theta_m \sim \mathcal{O}(1)$ and the integral $I_+ \rightarrow 0$ rapidly. At lower frequencies, such as those probed by the pulsars for example, [38], the effect of the extra dimensions essentially washes out, and at this level, the (super)string kinks will simply contribute to the general stochastic GW background, [33].

To sum up, although we find a significant suppression of the GWB signal from kinks due to extra dimensions, the overall suppression is much less than that for cusps. Thus, although the kink signal is weaker than that of the cusp, its independence of the number of extra dimensions, n , could lead to it being important, particularly if n is large. If, in addition, there are other kinematical effects driving kink formation, then the outlook for detection at LIGO becomes extremely optimistic over a wide range of (super)string tensions (although the current bounds, [39], do need to be revisited!). Clearly it is well worth a more detailed and comprehensive modelling of kinks on networks to give a definitive computation of the gravitational wave signal.

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